New Results on Antenna Arraying: Part 1

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Baseband combining with and without combined carrier referencing for antenna arrays are compared under two scenarios for the Voyager 2 Uranus encounter. The combined carrier reference scheme is estimated to outperform the baseband-only scheme by less than 0.3 dB E_b/N_0 at a bit error probability of 5×10^{-3}. These results were attained both with mathematical modeling and software Viterbi decoder simulations.

I. Introduction

Two methods have been proposed for the combining of signals received by an array of several antennas from deep space missions. The first, and by far the easier of the two, is known as "baseband-only combining." In this scheme, the receiver for each antenna heterodynes the incoming signal separately and the resulting baseband signals are simply added together. Since each receiver has its own phase lock loop for determining a reference phase for heterodyning, statistically independent phase errors (sometimes called "tracking errors") are introduced. This fact makes baseband-only combining awkward to model mathematically, and therefore its performance is difficult to estimate confidently without the use of a computer.

The second scheme is called "baseband combining with combined carrier referencing." In this scheme a single phase reference is determined by a single phase lock loop which examines the combined incoming signal. This reference is then used in every receiver for heterodyning to baseband. The difficulty in this scheme is that extremely accurate clocks are needed to synchronize the receivers. The amount by which this scheme will outperform baseband-only combining at Voyager 2 Uranus encounter is the subject of this article.

Figures 1 and 2 show simplified block diagrams of the two arraying schemes previously described. The delays are shown to compensate for the geometry of the array.

For the Voyager 2 Uranus encounter, arrays consisting of one 64-m antenna and one or more 34-m antennas are being considered. Arrays with two and three 34-m antennas will be considered in this article. An important question is whether baseband-only combining, which is less expensive and easier to implement, will suffice for this encounter or whether the addition of combined carrier referencing will be necessary.

A detailed analysis of baseband combined arrays is presently being undertaken and will be published in Part 2 of this study. The results presented here represent only a small portion of the intended effort, but they already show that, for the Uranus encounter, at most 0.3 dB channel E_b/N_0 is gained by using combined carrier referencing.

II. Theory of Baseband-only Combining

Suppose there are n antennas with a total area of unity. (This normalization is convenient for modeling purposes.) Let the

i^{th} antenna have effective area w_i and let its phase lock loop signal-to-noise ratio (SNR) be ρ_i. Loop SNR is a measure of how well a phase lock loop can track the phase of a signal. It is known (Ref. 1) then that the i^{th} antenna's loop will generate phase errors ϕ_i according to the distribution

$$p_i(\phi_i) = \frac{e^{\rho_i \cos\phi_i}}{2\pi I_0(\rho_i)}. \qquad (1)$$

If a signal of (data) amplitude A is received by this array, the baseband combined signal will have amplitude

$$A' = \sum_{i=1}^{n} A w_i \cos\phi_i .$$

Hence, if the data signal-to-noise ratio of the array were equal to E_b/N_0, then the SNR seen at the output of the baseband combiner would be

$$\text{TOTSNR} = (E_b/N_0)\left(\sum_{i=1}^{n} w_i \cos\phi_i\right)^2. \qquad (2)$$

Let $x_i = \cos\phi_i$ and let

$$x = \sum_{i=1}^{n} w_i x_i .$$

Then, if the ϕ_i vary slowly compared to the data rate of the signal (high rate model), the random variable x is distributed according to

$$f(x) = \int_{-1}^{1} \cdots \int_{-1}^{1} f_1(x_1) \ldots f_{n-1}(x_{n-1})$$

$$\times f_n((x - w_1 x_1 \ldots - w_{n-1}x_{n-1})/w_n)$$

$$\times dx_n \ldots dx_1$$

where

$$f_i(x_i) = \frac{e^{\rho_i x_i}}{\pi I_0(\rho_i)\sqrt{1-x_i^2}}.$$

Voyager 2 uses a (7, 1/2) convolutional code for which the probability of bit error (p_{BIT}) is known as a function of the channel E_b/N_0. Let this function be denoted by $p_{BIT}(E_b/N_0)$. Then the overall bit error performance of the baseband combined array is given as a function of E_b/N_0 by

$$p_{ARRAY}(E_b/N_0) = \int_{-1}^{1} p_{BIT}\left(\frac{E_b}{N_0}x^2\right) f(x)\,dx .$$

III. The Relationship Between Loop SNR and Carrier Margin

The theory presented in Section II assumed knowledge of the loop SNRs of each receiver. In practice, carrier margin, and not loop SNR, is the known quantity. This section contains a method for computing loop SNR from carrier margin. Figure 4 shows the dependence of loop SNR on carrier margin for the expected receiver configurations at Uranus encounter.

The tracking loop SNR is given by

$$\rho = \frac{P_C}{N_0 B_L \Gamma}$$

where P_C is the carrier power, N_0 is the one-sided noise spectral density, B_L is the loop bandwidth of the receiver, and Γ is the loop's bandpass limiter suppression factor. This formula can be rewritten as

$$\rho = m\frac{2B_{LO}}{B_L \Gamma}$$

where m and B_{LO} are the carrier margin and the threshold loop bandwidth respectively. In the case that $2B_{LO} = 30$ Hz, B_L may be calculated from m by using Fig. 3.

The quantity Γ is often approximated by unity as it is always between 1 and 1.2. However, Γ can be better approximated in the following way. In Ref. 1 it is shown that

$$\Gamma \approx \frac{1+\rho_H}{0.862+\rho_H}$$

where ρ_H is the SNR of the receiver's bandpass hard limiter, and is given by

$$\rho_H = \frac{3m}{w_H \tau_2}$$

where w_H and τ_2 are known filter parameters. For the receivers that will be used at Uranus encounter, $2B_{LO} = 30$ Hz, $w_H = 4000$, and $\tau_2 = 0.05$. Figure 4 indicates the dependence of the loop SNR ρ on the carrier margin m for these receivers.

IV. Application of Mathematical Model

The theory developed in Sections II and III can now be used to estimate the performance of any array proposed for the Uranus encounter by analyzing the appropriate mathematical model. For example, consider an array consisting of one 64-m antenna and two 34-m antennas. Suppose also that, of the two 34-m antennas in the array, one is a "listen only" (LO) antenna, while the other is a "transmit and receive" (T/R) antenna. The carrier margin and loop SNR of the LO antenna are greater than those for the T/R antenna. A lower bound to the performance of the array with baseband-only combining is obtained by assuming that both 34-m antennas are T/R.

An upper bound to the performance of the same array with combined carrier referencing is obtained by the performance of a single large antenna whose area equals the sum of the areas of the individual antennas. The carrier margin of this large antenna is the sum of the individual carrier margins. Although the loop SNRs are not additive, the cumulative loop SNR can be computed using Fig. 4. For example, two antennas with combined carrier referencing, each having $\rho = 10$ dB, would perform no better than a single antenna with $\rho = 12$ dB.

The expected carrier margins of the 64-m, 34-m (LO), and 34-m (T/R) antennas at Uranus encounter are 16.8 dB, 12.2 dB, and 11.0 dB respectively. The corresponding loop SNRs according to Fig. 4, are 13.4 dB, 10.1 dB, and 9.5 dB. The performance of this array with combined carrier referencing is bounded above by the performance of a single antenna with $m = 18.9$ dB or $\rho = 14.9$ dB.

The weights w_i, discussed in Section II, are proportional to the carrier margins of each antenna. For the lower bound to the baseband-only array performance, $w_1 = 0.65$ and $w_2 = w_3 = 0.175$. Since the upper bound to the array with combined carrier referencing consists of only a single antenna, $w_1 = 1$ for it.

Figure 5 shows the results of mathematical modeling applied to these parameters. It is clear from this figure that the array with combined carrier referencing performs at most 0.3 dB better than the baseband-only combined array at $p_{BIT} = 5 \times 10^{-3}$. For comparison, a graph of the ideal performance (i.e., no tracking losses) of the Viterbi decoder is shown in Fig. 6. The bit error rate of the Viterbi decoder is taken to be 1/2 at low channel SNRs, because the current DSN decoders lose node synchronization in this region.

V. Computer Simulations of Two Array Scenarios

In addition to mathematically modeling antenna array behavior, computer simulations were performed using a software Viterbi decoder.

The effective SNR seen by the Viterbi decoder was periodically updated according to Eq. (2) with each ϕ_i being randomly generated according to Eq. (1). The results of these simulations for the array described in Section IV are shown in Fig. 7. Again the difference between baseband combining with and without combined carrier referencing is seen to be less than 0.3 dB. The overall performance indicated by the curves in Fig. 7 is better than that indicated in Fig. 5, because the software Viterbi decoder never loses node synchronization.

In the second scenario there is one 64-m antenna and three 34-m antennas. Of the three 34-m antennas, one is T/R and two are LO. The performance of this four-antenna array with and without combined carrier referencing was also simulated using the Viterbi software decoder. The results are shown in Fig. 8. The difference in performance between these two schemes for this array configuration is also seen to be at most 0.3 dB.

VI. Summary and Conclusions

It has been shown, both by mathematical modeling and by computer simulation, that a three-antenna array with combined carrier referencing will perform only 0.3 dB better than the same array with baseband-only combining. Similarly, computer simulations show that a four-antenna array with combined carrier referencing will perform at most 0.3 dB better than the same array with baseband-only combining. Since the difference between these two combining schemes is at most 0.3 dB, careful consideration should be given as to which scheme will be implemented for the Voyager 2 Uranus encounter.

Acknowledgment

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Reference

1. Lindsey, W. C., and Simon, M. K., *Telecommunications Systems Engineering*, Prentice-Hall Inc., N.J., 1973.

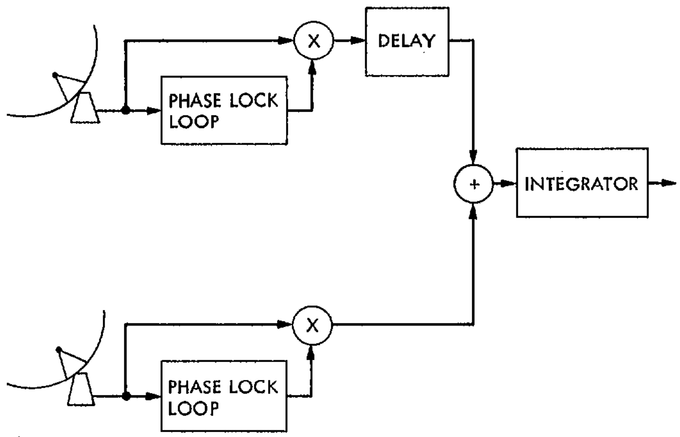

Fig. 1. Baseband-only combined arraying

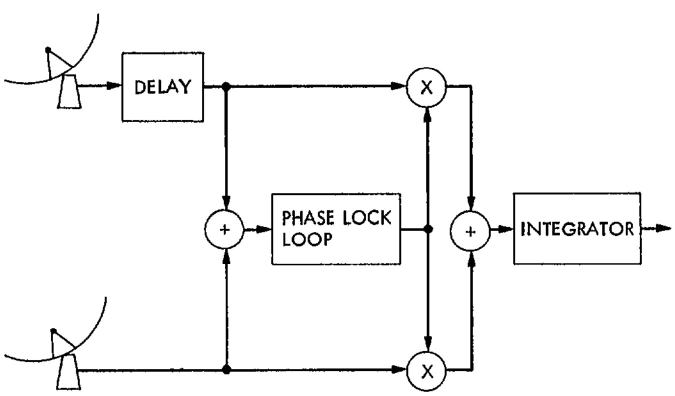

Fig. 2. Baseband combined arraying with combined carrier referencing

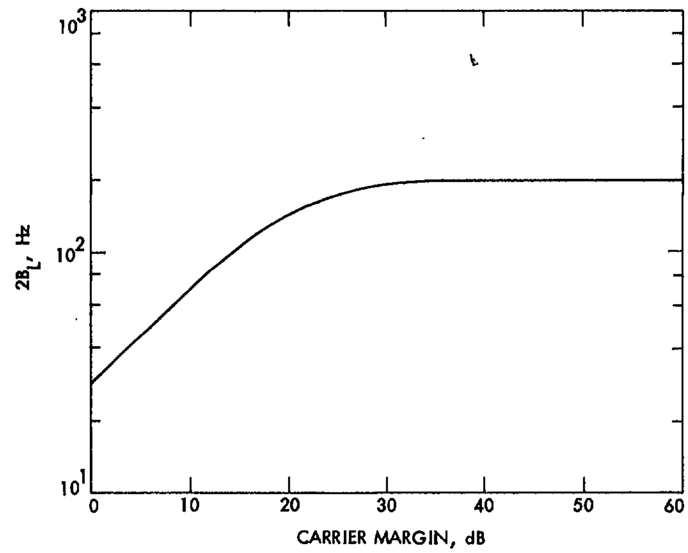

Fig. 3. Two-sided noise bandwidth as a function of carrier margin $2B_{LO} = 30$ Hz

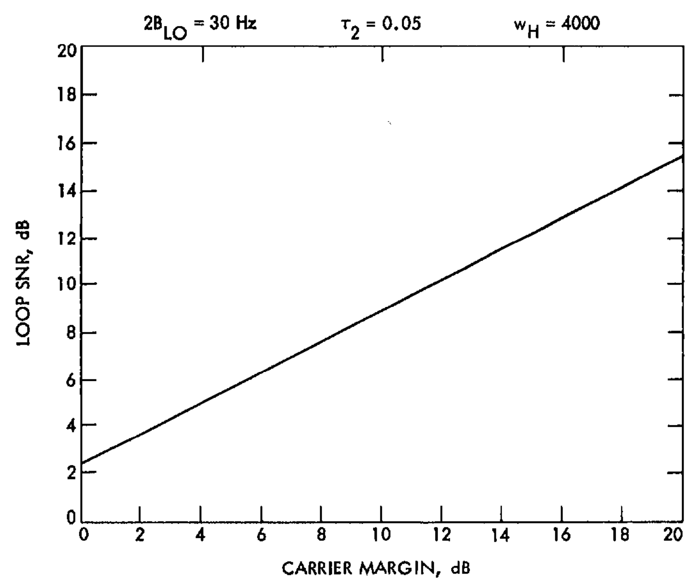

Fig. 4. Loop SNR as a function of carrier margin

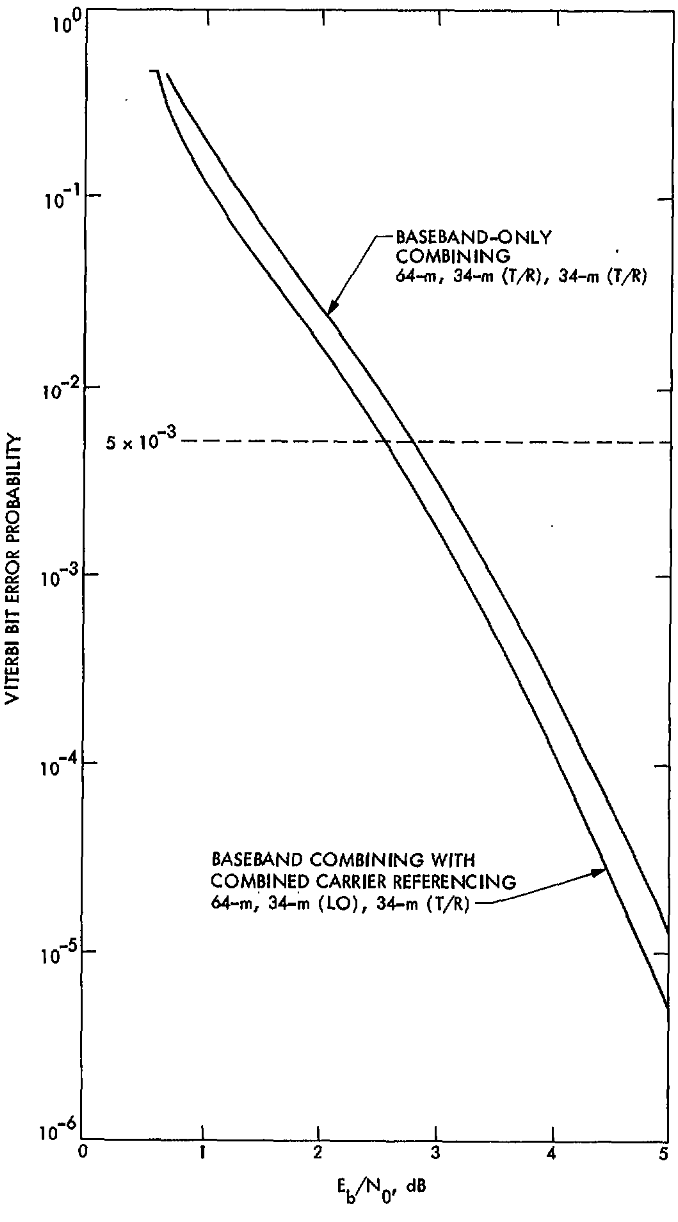

Fig. 5. Results of mathematical modeling for 64-m, 34-m, 34-m array at Uranus encounter

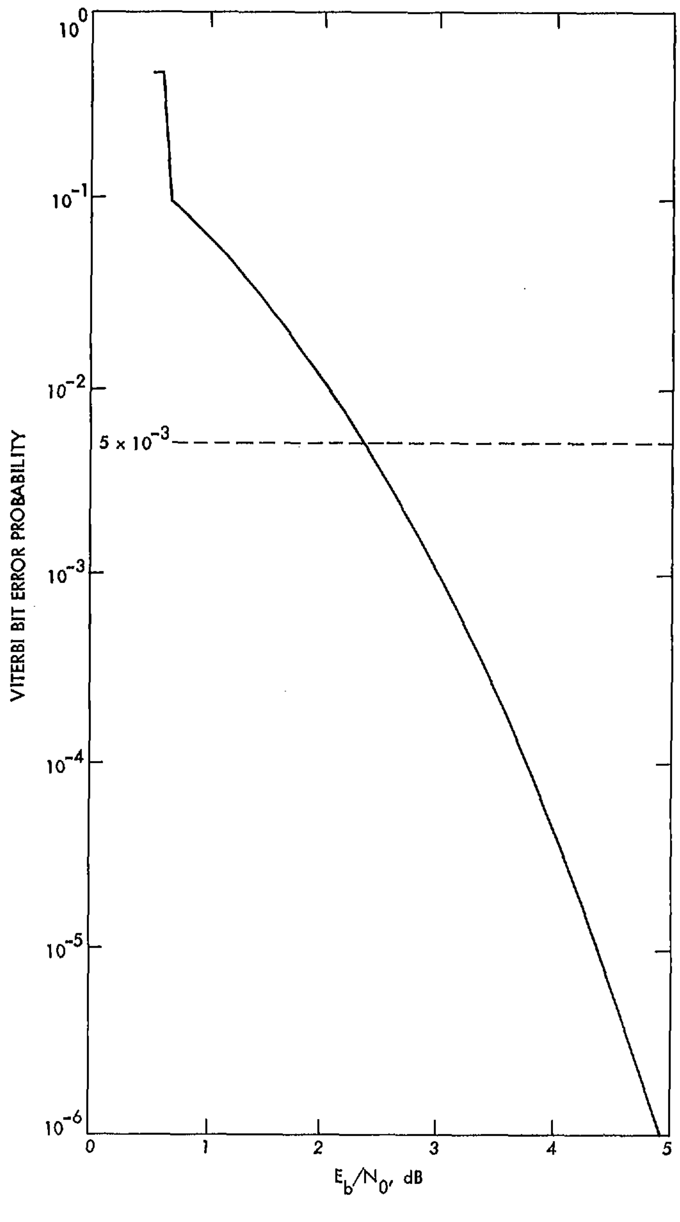

Fig. 6. Viterbi decoder performance without losses due to carrier tracking

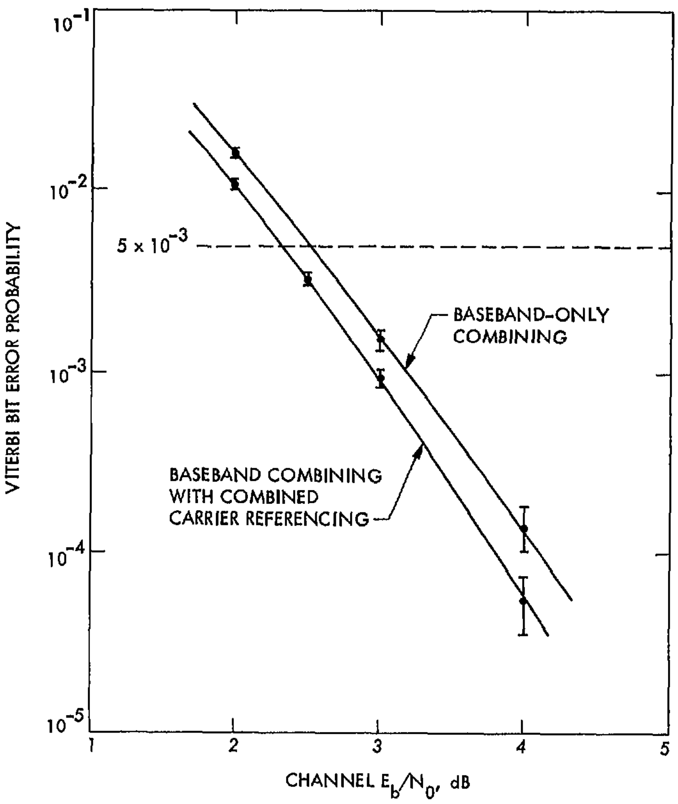

Fig. 7. Viterbi decoder simulations for 64-m, 34-m (LO), 34-m (T/R) array at Uranus encounter

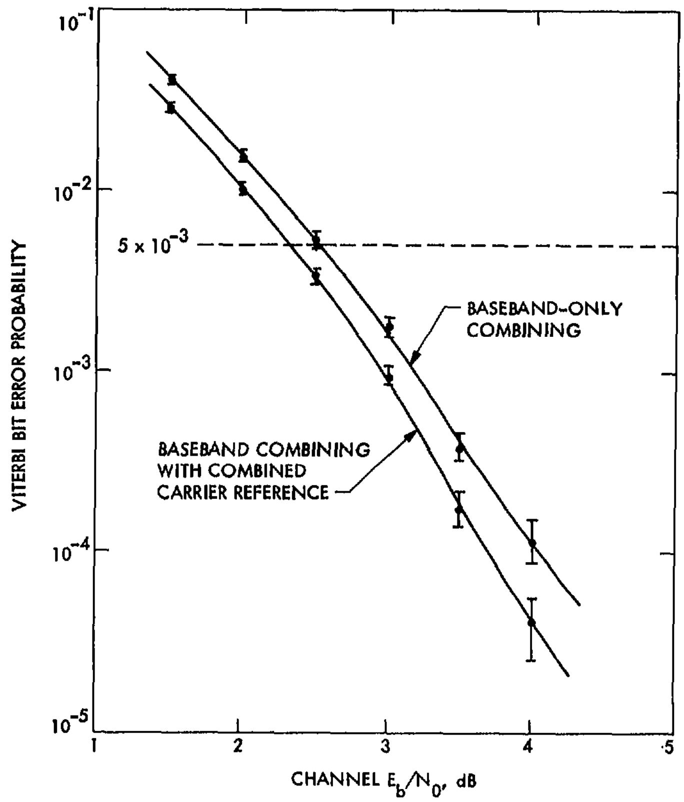

Fig. 8. Viterbi decoder simulations for 64-m, 34-m (LO), 34-m (LO), 34-m (T/R) array at Uranus encounter